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SOME CASES OF THE ELASTIC EQUILIBRIUM OF AN ANISOTROPIC PLATE WITH A NONCIRCULAR OPENING (PLANE PROBLEM)

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EDITED TRANSLATION

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SOME CASES OF THE ELASTIC EQUILIBRIUM OF AN ANISOTROPIC PLATE WITH A NONCIRCULAR OPENING (PLANE PROBLEM)

S. G. Lekhnitskiy

(Saratov)

Summary

In our report [1] there is given an approximate method for solving a plane problem of the theory of elasticity for an anisotropic plane weakened by an opening differing little from a circular opening and was the distribution of stresses close to an opening, having four axes of symmetry in case of elongation and bending moments was investigated. A comparison of sequential approximations for a plate made of veneer shows that the second approximation already assures sufficient accuracy, even in a case when the opening is close in form to a square with rounded corners, i.e., it differs in fact quite considerably from a circular one. The method can be spread over a quite wide class of openings which are considered as differing little from an elliptical opening and especially from a circular opening.

In this report there are given approximate solutions of a plane problem for an anisotropic plate with an opening close to an equilateral triangle with rounded corners and to a rectangle with twisted short sides, whereby expansion or bending by moments is discussed a general case of the distribution of external forces as well as special cases. Some general formulas which are used in finding approximate solutions for a plate with an opening are given preliminarily.

1. An approximate method of solving a plane problem for a plate with an opening differing little from an elliptical opening. Let us examine an elastic equilibrium of an infinite anisotropic plate with opening along whose edges are distributed forces which act in the middle plane (Fig. 1). Let the contour of the opening be given by equation

$$x = a(\cos\vartheta + \varepsilon\cos N\vartheta)$$
$$y = a(c\sin\vartheta - \varepsilon\sin N\vartheta) \tag{1.1}$$

where $0 < c \leq 1$, N is an integer different from one, and ε is low in absolute value as compared with one. Such an opening differs little from an elliptical opening with semiaxes a and ac, and a c = 1 – it differs little from a circular one. If c = 1, the opening will have N + 1 axis of symmetry; at a corresponding selection of ε it will be close to an equilateral triangle, at N = 3 it is close to a square, at N = 5 it is close to a hexagon, and so on. At c < 1 and N = 3 it is possible to select ε so that at a given length to width ratio the two sides of the opening will be practically rectilinear, and the two other sides will differ little from a semicircle.

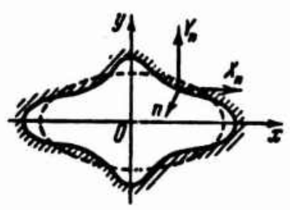

Fig. 1.

We will use ordinary designations for stress components and values connected with the elastic constants of the material (as in report [1]). Let us designate the projections of the external forces per unit of area by X_n, Y_n, and the arc of the contour opening by s. At first we will not assume the existence of any elements of elastic symmetry except the planes of an elastic symmetry parallel to the center xoy, and we will also further discuss the case of an orthotropic plate.

As it is known, the problem is reduced to determination in the area of a plate of two functions $\Phi_1(z_1)$ and $\Phi_2(z_2)$ of complex variables

$$z_1 = z + \lambda_1 \bar{z}, \quad z_2 = z + \lambda_2 \bar{z}$$

where

$$z = x + iy, \quad \bar{z} = x - iy, \quad \lambda_1 = \frac{1 + i\mu_1}{1 - i\mu_1}, \quad \lambda_2 = \frac{1 + i\mu_2}{1 - i\mu_2} \tag{1.2}$$

and μ_1 and μ_2 – complex parameters depending on elastic constants a_{ij}. These functions satisfy definite regularity conditions in the area of the plate, and on the contour of the opening at given external forces they satisfy the boundary conditions:

$$2\,\mathrm{Re}\,(\Phi_1 + \Phi_2) = f_1 + c_1, \quad 2\,\mathrm{Re}\,(\mu_1\Phi_1 + \mu_2\Phi_2) = f_2 + c_2. \tag{1.3}$$

Here

$$f_1 = \int_0^s Y_n ds, \qquad f_2 = -\int_0^s X_n ds. \tag{1.3a}$$

Re is the designation of the substantial part of the complex expression, c_1 and c_2 are constants which can be considered as arbitrary; integrals are taken along the arc of the contour of the opening from the initial point to the variable point (the opening is bypassed in a counterclockwise direction). Stress components are expressed through derivative functions of complex variables from the formulas

$$\begin{aligned} \sigma_x &= 2\,\mathrm{Re}\,[\mu_1^2(1 + \lambda_1)\Phi_1'(z_1) + \mu_2^2(1 + \lambda_2)\Phi_2'(z_2)] \\ \sigma_y &= 2\,\mathrm{Re}\,[(1 + \lambda_1)\Phi_1'(z_1) + (1 + \lambda_2)\Phi_2'(z_2)] \\ \tau_{xy} &= -2\,\mathrm{Re}\,[\mu_1(1 + \lambda_1)\Phi_1'(z_1) + \mu_2(1 + \lambda_2)\Phi_2'(z_2)]. \end{aligned} \tag{1.4}$$

For stress σ_ϑ close to an opening affecting the areas normal to its contour we have the formula

$$\sigma_\vartheta = \frac{2}{ds^2}\,\mathrm{Re}\,[(1 + \lambda_1)(dy - \mu_1 dx)^2\Phi_1'(z_1) + (1 + \lambda_2)(dy - \mu_2 dx)^2\Phi_2'(z_2)]. \tag{1.5}$$

The essence of the approximation method consists in the following [1].

Let us reflect the area of the plate, an infinite plane with a groove (1.1) on the exterior of a singular circle $|\zeta| = 1$. The reflecting function has the form of:

$$z = a\left(\frac{1 + c}{2}\zeta + \frac{1 - c}{2}\frac{1}{\zeta} + \frac{\varepsilon}{\zeta^N}\right). \tag{1.6}$$

The reflection will be mutually synonymous and conformal, if all roots of the derivative function on the right side of (1.6) will be smaller in absolute value than one. This imposes certain limitations on constants c and ε, which will be considered as satisfied. Expecially, at c = 1 it should be

$$N|\varepsilon|<1.$$

Let us introduce new complex variables ζ_1 and ζ_2:

$$\zeta_k=\frac{1+c}{2}\zeta+\frac{1-c}{2}\frac{1}{\zeta}+\lambda_k\left(\frac{1+c}{2}\bar{\zeta}+\frac{1-c}{2}\frac{1}{\bar{\zeta}}\right) \tag{1.7}$$

(k = 1, 2; $\bar{\zeta}$ is a value conjugate to ζ).

Functions Φ_1 and Φ_2 can be represented in the following form:

$$\begin{aligned}\Phi_1&=\Phi_{10}+\varepsilon\Phi_{11}+\cdots+\varepsilon^k\Phi_{1k}+\cdots\\ \Phi_2&=\Phi_{20}+\varepsilon\Phi_{21}+\cdots+\varepsilon^k\Phi_{2k}+\cdots.\end{aligned} \tag{1.8}$$

Φ_{1k} do not depend upon ε and are expressed by functions $f_{1k}(\zeta_1)$ of variable ζ_1 in this manner:

$$\begin{aligned}&\Phi_{10}=f_{10}(\zeta_1)\\ &\Phi_{11}=f_{11}(\zeta_1)+\left(\frac{1}{\zeta^N}+\frac{\lambda_1}{\bar{\zeta}^N}\right)f_{10}'(\zeta_1)\\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots\\ &\Phi_{1k}=f_{1k}(\zeta_1)+\left(\frac{1}{\zeta^N}+\frac{\lambda_1}{\bar{\zeta}^N}\right)f_{1,k-1}'(\zeta_1)+\cdots+\frac{1}{k!}\left(\frac{1}{\zeta^N}+\frac{\lambda_1}{\bar{\zeta}^N}\right)^k f_{10}^{(k)}(\zeta_1)\\ &\dots\dots\dots\dots\dots\end{aligned} \tag{1.9}$$

In a completely similar manner Φ_{2k} are expressed by functions $f_{2k}(\zeta_2)$.

For simplicity let us consider that the main vector of forces acting on the edges of the opening are equal to zero. Assuming that X_n, Y_n depend upon ε, let us break them down into series according to the stages of this parameter. The integrals which make up the right sides of boundary conditions (1.3) take the form:

$$\begin{aligned}f_1+c_1&=\sum_{k=0}\varepsilon^k\left[\alpha_{k0}+\sum_{m=1}^{\infty}(\alpha_{km}\sigma^m+\bar{\alpha}_{km}\sigma^{-m})\right]\\ f_2+c_2&=\sum_{k=0}\varepsilon^k\left[\beta_{k0}+\sum_{m=1}^{\infty}(\beta_{km}\sigma^m+\bar{\beta}_{km}\sigma^{-m})\right].\end{aligned} \tag{1.10}$$

Here α_{k0} and β_{k0} are arbitrary constants (substantial), α_{km} and β_{km} are given constants which depend on the law of the distribution of forces $\bar{\alpha}_{km}$ and $\bar{\beta}_{km}$ are conjugate values, and $\sigma = e^{i\vartheta}$. Boundary conditions (1.3) break down into series of pairs of conditions for functions Φ_{1k}, Φ_{2k}, and are accordingly conjugate to a number of degrees ε preserved in decompositions (1.8):

$$\begin{gathered}\Phi_{1k}+\Phi_{2k}+\bar{\Phi}_{1k}+\bar{\Phi}_{2k}=\alpha_{k0}+\sum_{m=1}^{\infty}(\alpha_{km}\sigma^{m}+\bar{\alpha}_{km}\sigma^{-m})\\ \mu_1\Phi_{1k}+\mu_2\Phi_{2k}+\bar{\mu}_1\bar{\Phi}_{1k}+\bar{\mu}_2\bar{\Phi}_{2k}=\beta_{k0}+\sum_{m=1}^{\infty}(\beta_{km}\sigma^{m}+\bar{\beta}_{km}\sigma^{-m})\\ (k=0,1,2\ldots).\end{gathered} \tag{1.11}$$

These conditions will be recorded in greater detail, thus (the arguments of function f_{1k} and f_{2k} are not written out):

$$\begin{gathered}f_{10}+f_{20}+\bar{f}_{10}+\bar{f}_{20}=\alpha_{00}+\sum_{m=1}^{\infty}(\alpha_{0m}\sigma^{m}+\bar{\alpha}_{0m}\sigma^{-m})\\ \mu_1 f_{10}+\mu_2 f_{20}+\bar{\mu}_1\bar{f}_{10}+\bar{\mu}_2\bar{f}_{20}=\beta_{00}+\sum_{m=1}^{\infty}(\beta_{0m}\sigma^{m}+\bar{\beta}_{0m}\sigma^{-m})\end{gathered} \tag{1.12.0}$$

$$\begin{gathered}f_{1k}+f_{2k}+\left(\frac{1}{\sigma^N}+\lambda_1\sigma^N\right)f'_{1,k-1}+\left(\frac{1}{\sigma^N}+\lambda_2\sigma^N\right)f'_{2,k-1}+\cdots\\ \cdots+\frac{1}{k!}\left(\frac{1}{\sigma^N}+\lambda_1\sigma^N\right)^k f_{10}^{(k)}+\frac{1}{k!}\left(\frac{1}{\sigma^N}+\lambda_2\sigma^N\right)^k f_{20}^{(k)}+\\ +\text{ conjug. values }=\alpha_{k0}+\sum_{m=1}^{\infty}(\alpha_{km}\sigma^{m}+\bar{\alpha}_{km}\sigma^{-m})\\ \mu_1 f_{1k}+\mu_2 f_{2k}+\mu_1\left(\frac{1}{\sigma^N}+\lambda_1\sigma^N\right)f'_{1,k-1}+\mu_2\left(\frac{1}{\sigma^N}+\lambda_2\sigma^N\right)f'_{2,k-1}+\cdots\\ \cdots+\frac{\mu_1}{k!}\left(\frac{1}{\sigma^N}+\lambda_1\sigma^N\right)^k f_{10}^{(k)}+\frac{\mu_2}{k!}\left(\frac{1}{\sigma^N}+\lambda_2\sigma^N\right)^k f_{20}^{(k)}+\\ +\text{ conjug. values }=\beta_{k0}+\sum_{m=1}^{\infty}(\beta_{km}\sigma^{m}+\bar{\beta}_{km}\sigma^{-m})\\ (k=1,2,3,\ldots).\end{gathered} \tag{1.12.k}$$

When ζ passes along the contour of a unit circle, variable z passes along the contour of the groove (1.1), and the variable

$$\zeta'=\frac{1+c}{2}\zeta+\frac{1-c}{2}\frac{1}{\zeta} \tag{1.13}$$

along the contour of an ellipse with semi-axes 1 and c. Consequently, functions f_{1k} and f_{2k} of variables $\zeta_1=\zeta'+\lambda_1\bar{\zeta}'$ and $\zeta_2=\zeta'+\lambda_2\bar{\zeta}'$ should be determined in an area which is an infinite plane with a groove in the form of the indicated ellipse. In that way, our problem

is reduced to a series of problems concerning the distribution of stresses in an infinite anisotropic plate with an elliptical opening, whose solutions are well known. Functions f_{1k} and f_{2k} have the form of*:

$$
\begin{aligned}
f_{1k} &= A_{k0} + \sum_{m=1}^{\infty} A_{km} \left[\frac{1 + \lambda_1 + c(1 - \lambda_1)}{\zeta_1 + \sqrt{\zeta_1^2 - (1 + \lambda_1)^2 + c^2(1 - \lambda_1)^2}} \right]^m \\
f_{2k} &= B_{k0} + \sum_{m=1}^{\infty} B_{km} \left[\frac{1 + \lambda_2 + c(1 - \lambda_2)}{\zeta_2 + \sqrt{\zeta_2^2 - (1 + \lambda_2)^2 + c^2(1 - \lambda_2)^2}} \right]^m .
\end{aligned}
\tag{1.14}
$$

At the points corresponding to points of the opening's contour (1.1) we have

$$
f_{1k} = A_{k0} + \sum_{m=1}^{\infty} A_{km} \sigma^{-m}, \qquad f_{2k} = B_{k0} + \sum_{m=1}^{\infty} B_{km} \sigma^{-m}. \tag{1.15}
$$

Constants A_{k0} and B_{k0} under given external forces do not affect the distribution of stresses and can be fixed arbitrarily, and A_{km} and B_{km} are determined from boundary conditions (1.12.0)–(1.12.k). From conditions (1.12.0) we find

$$
A_{0m} = \frac{\overline{\beta}_{0m} - \mu_2 \overline{\alpha}_{0m}}{\mu_1 - \mu_2}, \quad B_{0m} = -\frac{\overline{\beta}_{0m} - \mu_1 \overline{\alpha}_{0m}}{\mu_1 - \mu_2} \tag{1.16}
$$

and determine functions f_{10}, f_{20}. From the following conditions corresponding to $k = 1$ by order we determine A_{1m} and B_{1m} and functions f_{11}, f_{21}; these functions, together with f_{10} and f_{20}, will give a solution of the problem in first approximation. From conditions corresponding to $k = 2$ we find A_{2m}, B_{2m} and functions f_{12}, f_{22}, which, together with those prevously found, will determine a solution to our problem in the second approximation.

Thus any approximation can formally be plotted, preserving this or that number of terms in the breakdowns (1.8).

We will state that the problems of the convergence of the process of sequential approximations will not be touched here.

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*See [2], foreign page 90. In our book other designations were adopted.

The problem becomes somewhat complicated if the main vector of external forces does not equal zero. Then the integrals for (1.10), in addition to series, will contain the components $\alpha_k \ln \sigma$ and $\beta_k \ln \sigma$, and functions f_{1k} and f_{2k} are logarithmic terms.

2. *Some general formulae.* As was shown, coefficients A_{km} and B_{km} of functions f_{1k} and f_{2k} will be determined from boundary conditions (1.11) or (1.12.0)-(1.12.k). Under these conditions, in addition to boundary values of the functions themselves, these derivatives are figured

$$\frac{1}{n!}\left(\frac{1}{\sigma^N} + \lambda_1\sigma^N\right)^n f_{1k}^{(n)}, \quad \frac{1}{n!}\left(\frac{1}{\sigma^N} + \lambda_2\sigma^N\right)^n f_{2k}^{(n)}. \tag{2.1}$$

In the general case these expressions are represented in the form of sums of an infinitely large number of terms with negative degrees of σ and, in addition, they contain terms with positive degrees of σ where number, of course, also depends on n and N.

Let us indicate the general formulas for the derivatives of (2.1) at arbitrary values of n and N and $c \leq 1$, and also for certain particular values of these parameters. Let us introduce the designations:

$$\omega_k = \frac{2}{1+c+\lambda_k(1-c)}, \quad \alpha_k = \frac{1-c+\lambda_k(1+c)}{1+c+\lambda_k(1-c)}, \quad \beta_k = \omega_k\lambda_k \quad (k = 1,2). \tag{2.2}$$

Let us note that everywhere $|\alpha_k| < 1$.

By knowing how the boundary value of function f_{1k} is expressed [see (1.15)] we will easily find the boundary values of the first derivative and the derivatives of higher orders of that function using the formulas

$$\begin{gathered} f_{1k}' = \frac{df_{1k}}{d\sigma} : \frac{d\zeta_1}{d\sigma} = \omega_1 \frac{df_{1k}}{d\sigma} : \left(1 - \frac{\alpha_1}{\sigma^2}\right) = \omega_1 \frac{df_{1k}}{d\sigma}\left(1 + \frac{\alpha_1}{\sigma^2} + \frac{\alpha_1^2}{\sigma^4} + \cdots\right) \\ f_{1k}^{(n)} = \omega_1 \frac{df_{1k}^{(n-1)}}{d\sigma}\left(1 + \frac{\alpha_1}{\sigma^2} + \frac{\alpha_1^2}{\sigma^4} + \cdots\right). \end{gathered} \tag{2.3}$$

The final results are reduced to the following

$$\frac{1}{n!}\left(\frac{1}{\sigma^N}+\lambda_1\sigma^N\right)^n f_{1k}{}^{(n)} = \sum_{m=0}^{Nn-n-1} A^n_{k,-m}\sigma^m + \sum_{m=1}^{\infty} A^n_{km}\sigma^{-m} \tag{2.4}$$

$$(k=0,\ 1,\ 2,\ 3,\ \ldots,\ n=1,\ 2,\ 3,\ \ldots,\ N=2,\ 3,\ \ldots).$$

The coefficients at positive and negative degrees of σ depend on A_{km}, the coefficients of function f_{1k}. Let us indicate the general formula by which it is possible to determine A^n_{km}, if the necessary stipulation is made:

$$A^n_{km} = \sum_{i=1} A_{k,m+Nn-n+2-2i}\, g^n_{mi}(\lambda_1, \alpha_1). \tag{2.5}$$

Here g^n_{mi} is an integral polynomial with respect to λ_1 and α_1 plotted in the following manner:

$$\begin{gathered}
g^n_{mi}(\lambda_1,\alpha_1) = \frac{(-1)^n\omega_1{}^n}{(n-1)!\,n!}(m+Nn-n+2-2i)\alpha_1^{i-Nn-1}\times\\
\times[(m+Nn-n-i+2)(m+Nn-n-i+3)\ldots\\
\ldots(m+Nn-i)\cdot i(i+1)\ldots(i+n-2)(\lambda_1\alpha_1{}^N)^n+\\
+n(m+Nn-n-i+2-N)(m+Nn-n-i+3-N)\ldots\\
\ldots(m+Nn-i-N)(i-N)(i+1-N)\ldots(i+n-2-N)(\lambda_1\alpha_1{}^N)^{n-1}+\\
+\binom{n}{2}(m+Nn-n-i+2-2N)(m+Nn-n-i+3-2N)\ldots\\
\ldots(m+Nn-i-2N)(i-2N)(i+1-2N)\ldots(i+n-2-2N)(\lambda_1\alpha_1{}^N)^{n-2}+\ldots\\
\ldots+\binom{n}{2}(m-n-i+2+2N)(m-n-i+3+2N)\ldots\\
\ldots(m-i+2N)(i-Nn+2N)(i-Nn+2N+1)\ldots\\
\ldots(i-Nn+n+2N-2)(\lambda_1\alpha_1{}^N)^2+\\
+n(m-n-i+2+N)(m-n-i+3+N)\ldots\\
\ldots(m-i+N)(i-Nn+N)(i-Nn+N+1)\ldots\\
\ldots(i-Nn+n+N-2)\lambda_1\alpha_1{}^N+\\
+(m-n-i+2)(m-n-i+3)\ldots(m-i)\cdot(i-Nn)(i-Nn+1)\ldots\\
\ldots(i-Nn+n-2)]
\end{gathered} \tag{2.6}$$

where $\binom{n}{2}$, $\binom{n}{3}$, etc., are binomial factors.

Formulas (2.5)-(2.6) may be used in the following manner. On making a summation in (2.5) we have to reject all terms containing A_{km} with a negative second subscript (since f_{1k} breaks down only in negative degrees of σ, and consequently, all $A_{k,-m} = 0$). Determining g^n_{mi} formula (2.6) we must discard all the terms with negative degrees of α_1 if such appear at the given values of n and N. From that stipulation from formulas (2.5)-(2.6) is possible to find A^n_{km} for any

(integral) values k, n, m, including negative values of m, also.

If $c = 1$, then $\omega_k = 1$, $\alpha_k = \beta_k = \lambda_k$ $(k = 1, 2)$.

Let us introduce the expressions of the first three products (2.1) for two openings which are characterized by the parameters: 1) $c = 1$, $N = 2$, and 2) $c < 1$, $N = 3$. Henceforth, for brevity we shall call the opening of the first type "triangular" and the opening of the second type "oval."

1. The triangular opening

$$\left(\frac{1}{\sigma^3} + \lambda_1\sigma^2\right) f_{1k}' = -A_{k1}\lambda_1 + \sum_{m=1}^{\infty} A_{km}^1\sigma^{-m}$$

$$\frac{1}{2!}\left(\frac{1}{\sigma^3} + \lambda_1\sigma^2\right)^2 f_{1k}'' = A_{k1}\lambda_1^2\sigma + 3A_{k2}\lambda_1^2 + \sum_{m=1}^{\infty} A_{km}^2\sigma^{-m} \tag{2.7}$$

$$\frac{1}{3!}\left(\frac{1}{\sigma^3} + \lambda_1\sigma^2\right)^3 f_{1k}''' = -A_{k1}\lambda_1^3\sigma^2 - 4A_{k2}\lambda_1^3\sigma - (6A_{k1}\lambda_1^4 + 10A_{k3}\lambda_1^3) + \sum_{m=1}^{\infty} A_{km}^3\sigma^{-m}.$$

Here

$$A_{km}^1 = -\sum_{i=1}^{\infty} A_{k,m+3-2i}(m+3-2i)\lambda_1^{i-3}(\lambda_1^2 + 1)$$

$$A_{km}^2 = \frac{1}{2}\sum_{i=1}^{\infty} A_{k,m+4-2i}(m+4-2i)\lambda_1^{i-5}[(m+4-i)i\lambda_1^4 + 2(m+2-i)(i-2)\lambda_1^2 + (m-i)(i-4)] \tag{2.8}$$

$$A_{km}^3 = -\frac{1}{12}\sum_{i=1}^{\infty} A_{k,m+5-2i}(m+5-2i)\lambda_1^{i-7}[(m+5-i)\times (m+6-i)i(i+1)\lambda_1^6 + 3(m+3-i)(m+4-i)(i-2)(i-1)\lambda_1^4 + 3(m+1-i)(m+2-i)(i-4)(i-3)\lambda_1^2 + (m-1-i)(m-i)(i-6)(i-5)].$$

2. The oval opening

$$\left(\frac{1}{\sigma^3} + \lambda_1\sigma^2\right) f_{1k}' = -A_{k1}\beta_1\sigma - 2A_{k2}\beta_1 + \sum_{m=1}^{\infty} A_{km}^1\sigma^{-m}$$

$$\frac{1}{2!}\left(\frac{1}{\sigma^3} + \lambda_1\sigma^2\right)^2 f_{1k}'' = A_{k1}\beta_1^2\sigma^2 + 3A_{k2}\beta_1^2\sigma^2 + (3A_{k1}\alpha_1 + 6A_{k3})\beta_1^2\sigma + (8A_{k2}\alpha_1 + 10A_{k4})\beta_1^2 + \sum_{m=1}^{\infty} A_{km}^2\sigma^{-m} \tag{2.9}$$

$$\frac{1}{3!}\left(\frac{1}{\sigma^3}+\lambda_1\sigma^3\right)^3 f_{1k}''' = -A_{k1}\beta_1^2\sigma^6 - 4A_{k2}\beta_1^2\sigma^4 - (6A_{k1}\alpha_1 + 10A_{k3})\beta_1^2\sigma^3 - \\ -(20A_{k3}\alpha_1 + 20A_{k4})\beta_1^2\sigma^2 - (20A_{k1}\alpha_1^4 + 45A_{k3}\alpha_1 + \\ + 35A_{k5})\beta_1^2\sigma - (60A_{k3}\alpha_1^2 + 84A_{k4}\alpha_1 + 56A_{k6})\beta_1^3 + \sum_{m=1}^{\infty} A_{km}^3\sigma^{-m}.$$

Here

$$\begin{aligned} A_{km}^1 &= -\omega_1 \sum_{i=1} A_{k,m+4-2i}(m+4-2i)\alpha_1^{i-4}(\lambda_1\alpha_1^2+1) \\ A_{km}^2 &= \frac{\omega_1^2}{2}\sum_{i=1} A_{k,m+6-2i}(m+6-2i)\alpha_1^{i-7}[(m+6-i)i(\lambda_1\alpha_1^3)^2 + \\ &\quad + 2(m+3-i)(i-3)\lambda_1\alpha_1^3 + (m-i)(i-6)] \\ A_{km}^3 &= -\frac{\omega_1^3}{12}\sum_{i=1} A_{k,m+8-2i}(m+8-2i)\alpha_1^{i-10}[(m+8-i)(m + \\ &\quad + 9-i)i(i+1)(\lambda_1\alpha_1^3)^3 + \\ &\quad + 3(m+5-i)(m+6-i)(i-3)(i-2)(\lambda_1\alpha_1^3)^2 + \\ &\quad + 3(m+2-i)(m+3-i)(i-6)(i-5)\lambda_1\alpha_1^3 + \\ &\quad + (m-1-i)(m-i)(i-9)(i-8)]. \end{aligned} \tag{2.10}$$

The expressions containing values of the derivatives of function f_{2k} are determined from similar formulas derived from (2.3)-(2.10) by substituting A, α_1, β_1, λ_1, and ω_1 with the values B, α_2, β_2, λ_2, and ω_2.

In order to compute the stress of σ_ϑ at the edge of the opening it is necessary to find the boundary values of products Φ_1' and Φ_2'. If the exact values of functions Φ_1 and Φ_2 are known, the boundary values of their products will be obtained by the formula

$$\Phi_k' = \frac{d\Phi_k}{d\vartheta} : \frac{dz_k}{d\vartheta} = \frac{1}{a(1+\lambda_k)}\frac{d\Phi_k}{d\vartheta}\frac{1}{-(\sin\vartheta+\varepsilon N\sin N\vartheta)+\mu_k(c\cos\vartheta-\varepsilon N\cos N\vartheta)} \quad (k=1,2). \tag{2.11}$$

In the cases discussed below we determine only the approximate values of functions Φ_1 and Φ_2. Substituting same in (2.11) we will obtain approximate values of products Φ_1' and Φ_2' on the contour of the opening.

We will also introduce the designations:

$$\frac{d\Phi_1}{d\vartheta} = \varphi_1 + i\psi_1, \qquad \frac{d\Phi_2}{d\vartheta} = \varphi_2 + i\psi_2 \tag{2.12}$$

$$A = c\cos\vartheta - \varepsilon N\cos N\vartheta, \qquad B = \sin\vartheta + \varepsilon N\sin N\vartheta \\ C^2 = A^2 + B^2. \tag{2.13}$$

Let us write the formula for stresses σ_ϑ at the edge of the opening as:

$$\sigma_\vartheta = \frac{2}{aC^2}\,\mathrm{Re}\left[(\varphi_1 + i\psi_1)\frac{(A+\mu_1 B)^2}{\mu_1 A - B} + (\varphi_2 + i\psi_2)\frac{(A+\mu_2 B)^2}{\mu_2 A - B}\right]. \qquad (2.14)$$

Tracing the action of partial loads, we consider an orthotropic plate in which the complex parameters are purely imaginary and unequal: $\mu_1 = \beta_1$, $\mu_2 = \delta_1$, and consequently, λ_1 and λ_2 are substantial numbers smaller than one in absolute value:

$$\lambda_1 = \frac{1-\beta}{1+\beta}, \qquad \lambda_2 = \frac{1-\delta}{1+\delta}. \qquad (2.15)$$

Henceforth, such a plate is simply called "orthotropic." The final formula for σ_ϑ in such a plate after the separation of a substantial part in (2.14) takes the form:

$$\begin{aligned}\sigma_\vartheta = \frac{2}{aLC^2}\{&\varphi_1 B[A^4(2\beta^2-1)\delta^2 + A^2B^2(\beta^2\delta^2 + 2\beta^2 - 1) + B^4\beta^2] + \\ &+ \varphi_2 B[A^4(2\delta^2-1)\beta^2 + A^2B^2(\beta^2\delta^2 + 2\delta^2 - 1) + B^4\delta^2] + \\ &+ \psi_1 A\beta[A^4\delta^2 + A^2B^2(-\beta^2\delta^2 + 2\delta^2 + 1) + B^4(2-\beta^2)] + \\ &+ \psi_2 A\delta[A^4\beta^2 + A^2B^2(-\beta^2\delta^2 + 2\beta^2 + 1) + B^4(2-\delta^2)]\}.\end{aligned} \qquad (2.16)$$

Here and henceforth

$$L = (A^2\beta^2 + B^2)(A^2\delta^2 + B^2). \qquad (2.17)$$

From formulas found for an orthotropic plate with purely imaginary values of μ_1 and μ_2, we can easily obtain formulas for two other cases by means of the limiting process. when μ_1 and μ_2 are purely imaginary and equal and when they are complex.

Here let us introduce expressions for certain constant and variable values depending upon β and δ, which will be a part of the formulas for stresses in particular cases:

$$\begin{gathered} g = \frac{8(1-\beta\delta)}{(1+\beta)^2(1+\delta)^2} \\ h = \frac{2}{(1+\beta)^2(1+\delta)^2}[1 + 2(\beta+\delta)(\beta\delta - 1) + \beta^2 - 4\beta\delta + \delta^2 + \beta^2\delta^2] \\ k = \frac{4}{(1+\beta)(1+\delta)}, \qquad l = \frac{2(1-\beta-\delta-\beta\delta)}{(1+\beta)(1+\delta)} \\ a_{11} = \frac{\beta_1\beta - \beta_2\delta}{\beta-\delta}, \qquad b_{11} = \frac{\beta_1 - \beta_2}{\beta-\delta}, \qquad c_{11} = \frac{\beta_1\delta - \beta_2\beta}{\beta-\delta} \\ a_{21} = a_{11}^2 + \beta\delta b_{11}^2 + 2\frac{\alpha_1\beta_1^2\beta - \alpha_2\beta_2^2\delta}{\beta-\delta}, \qquad b_{21} = (\beta+\delta)b_{11}^2 + 2\frac{\alpha_1\beta_1^2 - \alpha_2\beta_2^2}{\beta-\delta} \end{gathered} \qquad (2.18)$$

$$c_{21} = c_{11}^2 + \beta\delta b_{11}^2 + 2\frac{a_1\beta_1^2\delta - a_2\beta_2^2\beta}{\beta - \delta} \tag{2.19}$$

$$a_{23} = \frac{\beta_1^2\beta - \beta_2^2\delta}{\beta - \delta}, \qquad b_{23} = \frac{\beta_1^2 - \beta_2^2}{\beta - \delta}, \qquad c_{23} = \frac{\beta_1^2\delta - \beta_2^2\beta}{\beta - \delta}$$

$$D^4 = -A^4\beta\delta + A^2B^2(1 - 2\beta\delta - \beta^2\delta^2) + B^4(2 - \beta\delta - \beta^2 - \delta^2) \tag{2.20}$$

$$E^4 = -B^4\beta\delta + A^2B^2(\beta^2\delta^2 - 2\beta\delta - 1) + A^4(2\beta^2\delta^2 - \beta\delta - \beta^2 - \delta^2).$$

For an isotropic plate

$$\beta = \delta = 1, \quad g = h = 0, \quad k = 1, \quad l = -1, \quad a_{11} = b_{11} = c_{11} = -\frac{1}{1+c}$$

$$a_{21} = b_{21} = c_{21} = \frac{2}{(1+c)^2}, \qquad a_{23} = b_{23} = c_{23} = 0. \tag{2.21}$$

We illustrate the obtained results by calculations for an orthotropic plate in which the main elastic constants (i.e., the constants found for the main directions of elasticity) have the following values: Young moduli, 1.2×10^5 kg/cm^2 and 0.6×10^5 kg/cm^2; Poisson factors, 0.071 and 0.036, the shear modulus, 0.07×10^5 kg/cm^2. These values of elastic constants were obtained for one of the veneer types*. For brevity we also call a plate with such elastic constants simply a veneer plate. If the direction of the x axis corresponds to the direction for which the Young modulus has its maximum value ($E_x = E_{max}$, $E_y = E_{min}$), we get: $\mu_1 = 4.11i$, $\mu_2 = 0.343i$, i.e., $\beta = 4.11$, $\delta = 0.343$. If the direction of the x axis corresponds to the direction for which the Young modulus is lowest ($E_x = E_{min}$, $E_y = E_{max}$) then $\mu_1 + 0.243i$, $\mu_2 = 2.91i$, $\beta = 0.243$, and $\delta = 2.91$.

3. *Determining stresses in a plate with a triangular opening.* Let us introduce a solution to a plane problem for an infinite plate with an opening whose contour equation has the form:

$$x = a(\cos\vartheta + \varepsilon\cos 2\vartheta), \quad y = a(\sin\vartheta - \varepsilon\sin 2\vartheta) \tag{3.1}$$

and consequently, c = 1, and N = 2. At $\varepsilon = 0.25$ the opening differs little from an equilateral triangle with rounded corners, whereby the curvature at points in the centers of the sides equals zero. Let us assume that the forces X_n and Y_n are distributed along the edges of the opening so that their main vector is equal to zero (otherwise, the distribution can be completely arbitrary).

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*See [3], foreign page 133.

In wishing to solve the problem in their approximation, we should preserve in the third degrees of ε, which we rejected above the expressions for Φ_1 and Φ_2 (1.8). We have

$$\Phi_1 = \Phi_{10} + \varepsilon\Phi_{11} + \varepsilon^2\Phi_{12} + \varepsilon^3\Phi_{13}, \quad \Phi_2 = \Phi_{20} + \varepsilon\Phi_{21} + \varepsilon^2\Phi_{22} + \varepsilon^3\Phi_{23}. \tag{3.2}$$

In formulas (1.9) and (1.14) we should assume that c = 1 and N = 2. The expressions for functions f_{1k} and f_{2k} are simplified and take the form:

$$f_{1k} = A_{k0} + \sum_{m=1}^{\infty} A_{km}\left(\frac{2}{\zeta_1 + \sqrt{\zeta_1^2 - 4\lambda_1}}\right)^m, \quad f_{2k} = B_{k0} + \sum_{m=1}^{\infty} B_{km}\left(\frac{2}{\zeta_2 + \sqrt{\zeta_2^2 - 4\lambda_2}}\right)^m \tag{3.3}$$

where

$$\zeta_1 = \zeta + \lambda_1\bar{\zeta}, \qquad \zeta_2 = \zeta + \lambda_2\bar{\zeta}. \tag{3.4}$$

The conditions of (1.11) should be satisfied at the shape of the opening.

On the basis of formulas (1.9) and (2.7) we obtain the boundary values of functions Φ_{1k} and Φ_{2k}:

$$\Phi_{10} = A_0 + \sum_{m=1}^{\infty} A_{0m}\sigma^{-m}, \qquad \Phi_{20} = B_0 + \sum_{m=1}^{\infty} B_{0m}\sigma^{-m} \tag{3.5}$$

$$\Phi_{11} = A_1 + \sum_{m=1}^{\infty} (A_{1m} + A_{0m}^1)\sigma^{-m}, \quad \Phi_{21} = B_1 + \sum_{m=1}^{\infty} (B_{1m} + B_{0m}^1)\sigma^{-m} \tag{3.6}$$

$$\begin{gathered} \Phi_{12} = A_2 + A_{01}\lambda_1^2\sigma + \sum_{m=1}^{\infty} (A_{2m} + A_{1m}^1 + A_{0m}^2)\sigma^{-m} \\ \Phi_{22} = B_2 + B_{01}\lambda_2^2\sigma + \sum_{m=1}^{\infty} (B_{2m} + B_{1m}^1 + B_{0m}^2)\sigma^{-m} \end{gathered} \tag{3.7}$$

$$\begin{aligned} \Phi_{13} &= A_3 + (A_{11}\lambda_1^2 - 4A_{02}\lambda_1^3)\sigma - A_{01}\lambda_1^2\sigma^2 + \\ &\quad + \sum_{m=1}^{\infty} (A_{3m} + A_{2m}^1 + A_{1m}^2 + A_{0m}^3)\sigma^{-m} \\ \Phi_{23} &= B_3 + (B_{11}\lambda_2^2 - 4B_{02}\lambda_2^3)\sigma - B_{01}\lambda_2^2\sigma_2^2 + \\ &\quad + \sum_{m=1}^{\infty} (B_{3m} + B_{2m}^1 + B_{1m}^2 + B_{0m}^3)\sigma^{-m}. \end{aligned} \tag{3.8}$$

For constant components having no effect on the distribution of stresses, the abbreviated designations A_0, B_0, ..., B_3 were adopted.

From the boundary conditions of (1.11) we obtain systems of equations which are satisfied by the factors A_{km} and B_{km}. Let us introduce equations for $k = 0$, 1 and 2:

$$\begin{aligned} A_{0m} + B_{0m} &= \bar{\alpha}_{0m} \\ \mu_1 A_{0m} + \mu_2 B_{0m} &= \bar{\beta}_{0m} \end{aligned} \tag{3.9}$$

$$\begin{aligned} (A_{1m} + A_{0m}{}^1) + (B_{1m} + B_{0m}{}^1) &= \bar{\alpha}_{1m} \\ \mu_1 (A_{1m} + A_{0m}{}^1) + \mu_2 (B_{1m} + B_{0m}{}^1) &= \bar{\beta}_{1m} \end{aligned} \tag{3.10}$$

$$\begin{aligned} (A_{21} + A_{11}{}^1 + A_{01}{}^2) + (B_{21} + B_{11}{}^1 + B_{01}{}^2) &= \bar{\alpha}_{21} - \bar{A}_{01}\bar{\lambda}_1{}^2 - \bar{B}_{01}\bar{\lambda}_2{}^2 \\ \mu_1 (A_{21} + A_{11}{}^1 + A_{01}{}^2) + \mu_2 (B_{21} + B_{11}{}^1 + B_{01}{}^2) &= \bar{\beta}_{21} - \bar{A}_{01}\bar{\mu}_1\bar{\lambda}_1{}^2 - \bar{B}_{01}\bar{\mu}_2\bar{\lambda}_2{}^2 \end{aligned} \tag{3.11.1}$$

$$\begin{aligned} (A_{2m} + A_{1m}{}^1 + A_{0m}{}^2) + (B_{2m} + B_{1m}{}^1 + B_{0m}{}^2) &= \bar{\alpha}_{2m} \\ \mu_1 (A_{2m} + A_{1m}{}^1 + A_{0m}{}^2) + \mu_2 (B_{2m} + B_{1m}{}^1 + B_{0m}{}^2) &= \bar{\beta}_{2m} \quad (m \geqslant 2), \end{aligned} \tag{3.11.m}$$

Solving them, we find

$$A_{0m} = \frac{\bar{\beta}_{0m} - \mu_2\bar{\alpha}_{0m}}{\mu_1 - \mu_2}, \quad B_{0m} = -\frac{\bar{\beta}_{0m} - \mu_1\bar{\alpha}_{0m}}{\mu_1 - \mu_2} \tag{3.12}$$

$$A_{1m} + A_{0m}{}^1 = \frac{\bar{\beta}_{1m} - \mu_2\bar{\alpha}_{1m}}{\mu_1 - \mu_2}, \quad B_{1m} + B_{0m}^1 = -\frac{\bar{\beta}_{1m} - \mu_1\bar{\alpha}_{1m}}{\mu_1 - \mu_2} \tag{3.13}$$

$$\begin{aligned} A_{21} + A_{11}{}^1 + A_{01}{}^2 &= \frac{\bar{\beta}_{21} - \mu_2\bar{\alpha}_{21}}{\mu_1 - \mu_2} + \frac{\bar{A}_{01}\bar{\lambda}_1{}^2(\mu_2 - \bar{\mu}_1) + \bar{B}_{01}\bar{\lambda}_2{}^2(\mu_2 - \bar{\mu}_2)}{\mu_1 - \mu_2} \\ B_{21} + B_{11}{}^1 + B_{01}{}^2 &= -\frac{\bar{\beta}_{21} - \mu_1\bar{\alpha}_{21}}{\mu_1 - \mu_2} - \frac{\bar{A}_{01}\bar{\lambda}_1{}^2(\mu_1 - \bar{\mu}_1) + \bar{B}_{01}\bar{\lambda}_2{}^2(\mu_1 - \bar{\mu}_2)}{\mu_1 - \mu_2} \end{aligned} \tag{3.14.1}$$

$$\left.\begin{aligned} A_{2m} + A_{1m}{}^1 + A_{0m}{}^2 &= \frac{\bar{\beta}_{2m} - \mu_2\bar{\alpha}_{2m}}{\mu_1 - \mu_2} \\ B_{2m} + B_{1m}{}^1 + B_{0m}{}^2 &= -\frac{\bar{\beta}_{2m} - \mu_1\bar{\alpha}_{2m}}{\mu_1 - \mu_2} \end{aligned}\right\} (m > 2). \tag{3.14.m}$$

Finally, the boundary values of functions Φ_{10}, Φ_{11}, and Φ_{12} will be recorded in the following manner:

$$\Phi_{10} = A_0 + \sum_{m=1}^{\infty} \frac{\bar{\beta}_{0m} - \mu_2\bar{\alpha}_{0m}}{\mu_1 - \mu_2}\sigma^{-m} \tag{3.15}$$

$$\Phi_{11} = A_1 + \sum_{m=1}^{\infty} \frac{\bar{\beta}_{1m} - \mu_2\bar{\alpha}_{1m}}{\mu_1 - \mu_2}\sigma^{-m} \tag{3.16}$$

$$\Phi_{12} = A_2 + \sum_{m=1}^{\infty} \frac{\bar{\beta}_{2m} - \mu_2\bar{\alpha}_{2m}}{\mu_1 - \mu_2}\sigma^{-m} + A_{01}\lambda_1{}^2\sigma + \frac{\bar{A}_{01}\bar{\lambda}_1{}^2(\mu_2 - \bar{\mu}_1) + \bar{B}_{01}\bar{\lambda}_2{}^2(\mu_2 - \bar{\mu}_2)}{\mu_1 - \mu_2}\frac{1}{\sigma}. \tag{3.17}$$

Since the constant components are arbitrary in the right sides of boundary conditions (1.11), the constant components A_0, A_1, and A_2 (as well as B_0, B_1, B_2) remain arbitrary. For coefficients of functions Φ_{13} and Φ_{23} we get several more complex equations. Without

copying them let us show the expressions for the boundary value of function Φ_{13}:

$$\Phi_{13} = A_3 + \sum_{m=1}^{\infty} \frac{\bar{\beta}_{3m} - \mu_2\bar{\alpha}_{3m}}{\mu_1 - \mu_2}\sigma^{-m} + (A_{11}\lambda_1^2 - 4A_{02}\lambda_1^3)\sigma - A_{01}\lambda_1^3\sigma^2 +$$
$$+ \left[(\bar{A}_{11}\bar{\lambda}_1^2 - 4\bar{A}_{02}\bar{\lambda}_1^3)\frac{\mu_2 - \bar{\mu}_1}{\mu_1 - \mu_2} + (\bar{B}_{11}\bar{\lambda}_2^2 - 4\bar{B}_{02}\bar{\lambda}_2^3)\frac{\mu_2 - \bar{\mu}_2}{\mu_1 - \mu_2}\right]\frac{1}{\sigma} - \tag{3.18}$$
$$- \frac{\bar{A}_{01}\bar{\lambda}_1^3(\mu_2 - \bar{\mu}_1) + \bar{B}_{01}\bar{\lambda}_2^3(\mu_2 - \bar{\mu}_2)}{\mu_1 - \mu_2}\frac{1}{\sigma^2}.$$

We shall not derive the boundary values of functions Φ_{20}, Φ_{21}, Φ_{22}, and Φ_{23}; they are derived from (3.15)-(3.18) by replacing values A, B, λ_1, λ_2, μ_1, and μ_2 by the values B, A, λ_2, λ_1, μ_2, and μ_1 respectively.

Without derivation let us also introduce the boundary value of function Φ_{14}, which we need if we want to solve the problem in fourth approximation:

$$\Phi_{14} = A_4 + \sum_{m=1}^{\infty} \frac{\bar{\beta}_{4m} - \mu_2\bar{\alpha}_{4m}}{\mu_1 - \mu_2}\sigma^{-m} + (A_{21}\lambda_1^2 - 4A_{12}\lambda_1^3 + 10A_{01}\lambda_1^5 + 15A_{03}\lambda_1^4)\sigma +$$
$$+ (5A_{02}\lambda_1^4 - A_{11}\lambda_1^3)\sigma^2 + A_{01}\lambda_1^4\sigma^3 +$$
$$+ \left[(\bar{A}_{21}\bar{\lambda}_1^2 - 4\bar{A}_{12}\bar{\lambda}_1^3 + 10\bar{A}_{01}\bar{\lambda}_1^5 + 15\bar{A}_{03}\bar{\lambda}_1^4)\frac{\mu_2 - \bar{\mu}_1}{\mu_1 - \mu_2} + \right.$$
$$\left. + (\bar{B}_{21}\bar{\lambda}_2^2 - 4\bar{B}_{12}\bar{\lambda}_2^3 + 10\bar{B}_{01}\bar{\lambda}_2^5 + 15\bar{B}_{03}\bar{\lambda}_2^4)\frac{\mu_2 - \bar{\mu}_2}{\mu_1 - \mu_2}\right]\frac{1}{\sigma} + \tag{3.19}$$
$$+ \left[(5\bar{A}_{02}\bar{\lambda}_1^4 - \bar{A}_{11}\bar{\lambda}_1^3)\frac{\mu_2 - \bar{\mu}_1}{\mu_1 - \mu_2} + (5\bar{B}_{02}\bar{\lambda}_2^4 - \bar{B}_{11}\bar{\lambda}_2^3)\frac{\mu_2 - \bar{\mu}_2}{\mu_1 - \mu_2}\right]\frac{1}{\sigma^2} +$$
$$+ \frac{\bar{A}_{01}\bar{\lambda}_1^4(\mu_2 - \bar{\mu}_1) + \bar{B}_{01}\bar{\lambda}_2^4(\mu_2 - \bar{\mu}_2)}{\mu_1 - \mu_2}\frac{1}{\sigma^3}.$$

Let us determine the factors A_{11}, A_{12}, and A_{21} from (3.13) and (3.14) with the consideration of the first and second formulas (2.8). We will obtain

$$A_{11} = \frac{\bar{\beta}_{11} - \mu_2\bar{\alpha}_{11}}{\mu_1 - \mu_2} + 2A_{02}\lambda_1$$
$$A_{12} = \frac{\bar{\beta}_{12} - \mu_2\bar{\alpha}_{12}}{\mu_1 - \mu_2} + A_{01}\lambda_1^2 + 3A_{03}\lambda_1 \tag{3.20}$$

$$A_{21} = \frac{\bar{\beta}_{21} - \mu_2\bar{\alpha}_{21}}{\mu_1 - \mu_2} + \frac{\bar{A}_{01}\bar{\lambda}_1^2(\mu_2 - \bar{\mu}_1) + \bar{B}_{01}\bar{\lambda}_2^2(\mu_2 - \bar{\mu}_2)}{\mu_1 - \mu_2} +$$
$$+ 2A_{12}\lambda_1 - 4A_{02}\lambda_1^2 - 10A_{04}\lambda_1. \tag{3.21}$$

Coefficients B_{11}, B_{12}, and B_{21} are determined from analogous formulas which are derived from (3.20) and (3.21) by a simple rearrangement.

In order to calculate the stress σ_ϑ at the edge of the opening it is necessary to find derivatives of boundary values Φ_{1k} and Φ_{2k} according to and their real and imaginary parts φ_{1k}, φ_{2k}, ψ_{1k}, ψ_{2k}, and then use formula (2.14), or in case of an orthotropic plate, formula (2.16). In these formulas it is necessary to assume:

$$A = \cos\vartheta - 2\varepsilon\cos 2\vartheta,\ B = \sin\vartheta + 2\varepsilon\sin 2\vartheta,\ C^2 = 1 + 4\varepsilon^2 - 4\varepsilon\cos 3\vartheta. \qquad (3.22)$$

4. *The elongation of an orthotropic plate with a triangular opening.* Let us examine a rectangular orthotropic plate weakened in the center by a triangular opening and loaded by normal forces of intensity p and p' uniformly distributed over the sides.

We will solve the problem under the following assumptions: 1) the dimensions of the opening are small in comparison to the dimensions of the plate; 2) the main direction of elasticity (i.e., directions normal to the planes of elastic symmetry) are parallel to the sides of the plate, and the opening is cut out so that the direction of one of its three symmetrical axes coincides with the main axis. We take the center of the opening as the origin of coordinates and direct the x axis along the axis of symmetry parallel to the side of the plate. The equation of opening's contour has the form of (3.1).

We shall get the approximate solution of the problem, using a known method. Let σ_x°, σ_y°, and τ_{xy}° be the stress components in a plate without an opening. On line (3.1) there act forces with the projections:

$$X_n^\circ = \sigma_x^\circ \cos(n, x) + \tau_{xy}^\circ \cos(n, y), \quad Y_n^\circ = \tau_{xy}^\circ \cos(n, x) + \sigma_y^\circ \cos(n, y) \qquad (4.1)$$

n is the normal to curve (3.1),

$$\cos(n, x) = -\frac{dy}{ds}, \qquad \cos(n, y) = \frac{dx}{ds}. \qquad (4.2)$$

Let us superimpose a distribution of stresses in the infinite plate with the opening toward whose edge the forces

$$X_n = -X_n^\circ, \quad Y_n = -Y_n^\circ \qquad (4.3)$$

are applied on the stress distribution in the solid ("basic") plate.

This additional distribution will be found by means of functions Φ_{1k} and Φ_{2k} given in § 3. The stresses obtained as result of such superimposition will accurately satisfy the conditions on the contour of the opening, and they will tend to stresses in the solid plate in proportion to the removal from the opening. Thus, we will also formulate solutions for the case of bending by moments. Since all the formulas necessary for calculating the stresses have been derived in the preceding paragraphs, we, upon examining particular cases, will only introduce the formulas for stresses of coefficients $\bar{\alpha}_{km}$, $\bar{\beta}_{km}$, A_{0m}, and B_{0m} in a plate without an opening and a final formula for stress σ_ϑ at the edge of the opening (the third approximation) and at individual points.

Let us stop at two basic cases.

Case 1. Elongation of the opening in the direction of axis of symmetry (Fig. 2). If only forces p, distributed uniformly along two sides, are active we have:

$$\sigma_x^\circ = p, \qquad \sigma_y^\circ = \tau_{xy}^\circ = 0$$

$$X_n^\circ = p\cos(n, x) = -p\frac{dy}{ds} \tag{4.4}$$

$$Y_n^\circ = 0 \tag{4.5}$$

$$X_n = p\frac{dy}{ds}, \qquad Y_n = 0. \tag{4.6}$$

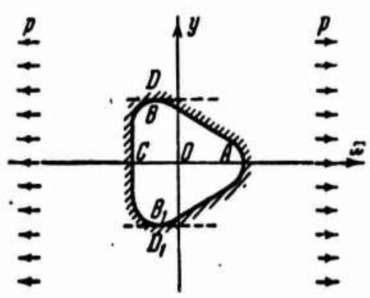

Fig. 2.

For functions f_1 and f_2 and coefficients $\bar{\alpha}$ and $\bar{\beta}$ we have an expression:

$$f_1 = 0, \qquad f_2 = -\int_0^s X_n\, ds = -py = \frac{pai}{2}\left[\sigma - \frac{1}{\sigma} + \varepsilon\left(-\sigma^2 + \frac{1}{\sigma^2}\right)\right] \tag{4.7}$$

$$\bar{\beta}_{01} = -\frac{pai}{2}, \quad \bar{\beta}_{12} = \frac{pai}{2} \tag{4.8}$$

other values of $\bar{\beta}_{km}$ and all values of $\bar{\alpha}_{km}$ are equal to zero;

$$A_{01} = -\frac{p\varepsilon}{2(\beta - \delta)}, \quad B_{01} = \frac{p\varepsilon}{2(\beta - \delta)}, A_{0m} = B_{0m} = 0 \quad (m \geq 2) \tag{4.9}$$

$$\begin{aligned} \sigma_\vartheta = p\frac{B^2}{C^2} + \frac{p}{LC^2}\{AD^4\cos\vartheta + BC^4(\beta+\delta)\sin\vartheta - \\ - 2\varepsilon[AD^4\cos 2\vartheta + BC^4(\beta+\delta)\sin 2\vartheta] + \\ + \varepsilon^2C^4(\beta+\delta)(Bh\sin\vartheta + A\beta\delta g\cos\vartheta) - \varepsilon^2BC^4(\beta+\delta)(lh - \\ - kg\beta\delta)\sin 2\vartheta - \varepsilon^2AC^4(\beta+\delta)\beta\delta[kh + gl + gk(\beta+\delta)]\cos 2\vartheta\}. \end{aligned} \tag{4.10}$$

Values A, B, and C^2 will be found by formulas (3.12), and values of D^4 will be found by (2.20).

At points $A(\vartheta = 0)$ and $C(\vartheta = \pi)$ we will obtain*

$$\sigma_A = \frac{p}{\beta\delta}\left\{-1 + \frac{\varepsilon^2}{1-2\varepsilon}(\beta+\delta)g - \frac{\varepsilon^2}{1-2\varepsilon}(\beta+\delta)[kh + gl + gk(\beta+\delta)]\right\} \tag{4.11}$$

$$\sigma_C = \frac{p}{\beta\delta}\left\{-1 + \frac{\varepsilon^2}{1+2\varepsilon}(\beta+\delta)g + \frac{\varepsilon^2}{1+2\varepsilon}(\beta+\delta)[kh + gl + gk(\beta+\delta)]\right\}. \tag{4.12}$$

Results of calculations for various orthotropic materials show that in these places where the curvature of the contour changes sharply (in regions of rounded corners near points B and B_1) the stress also changes sharply. It reaches maximum values close to points D and D_1, where the tangent to the contour is parallel to the acting forces (or that in the given case the same is parallel to the x axis).

For these points $\vartheta = \pm\vartheta_0$, whereby ϑ_0 will be determined from the equation

$$\cos\vartheta_0 - 2\varepsilon\cos 2\vartheta_0 = 0. \tag{4.13}$$

In particular, for $\varepsilon = 0.25$ we obtain: $\vartheta_0 = 111°30'$ at points D and D_1 the stress is determined by formula

$$\sigma_D = p\left\{1 + \frac{\beta+\delta}{C}[\sin\vartheta_0 - 2\varepsilon\sin 2\vartheta_0 + \varepsilon^2h\sin\vartheta_0 - \varepsilon^2(lh - kg\beta\delta)\sin 2\vartheta_0]\right\}. \tag{4.14}$$

*For stress σ_ϑ at given point M on the contour of the opening the designation σ_M is adopted here and henceforth.

For an isotropic plate

$$\beta = \delta = 1, \quad g = h = 0, \quad k = 1, \quad l = -1, \quad D^4 = -C^4, \quad L = C^4$$

and we obtain

$$\sigma_\vartheta = \frac{p}{C^2}\left(1 - 2\cos 2\vartheta + 4\varepsilon \cos\vartheta - 4\varepsilon^2\right). \tag{4.15}$$

It is interesting to mention that this formula, derived from approximate formula (4.10) is identical with an exact formula*.

Case 2. Elongation in a direction perpendicular to the axis of symmetry of the opening (Fig. 3):

$$\sigma_x{}^\circ = 0, \quad \sigma_y{}^\circ = p', \quad \tau_{xy}{}^\circ = 0 \tag{4.16}$$

$$\bar{\alpha}_{01} = \bar{\alpha}_{12} = -\frac{p'a}{2} \tag{4.17}$$

other values of $\bar{\alpha}_{km}$ and all values of $\bar{\beta}_{km}$ are equal to zero;

$$\begin{gathered} A_{01} = \frac{p'a\delta}{2(\beta - \delta)}, \qquad B_{01} = -\frac{p'a\beta}{2(\beta - \delta)} \\ A_{0m} = B_{0m} = 0 \qquad (m \geq 2). \end{gathered} \tag{4.18}$$

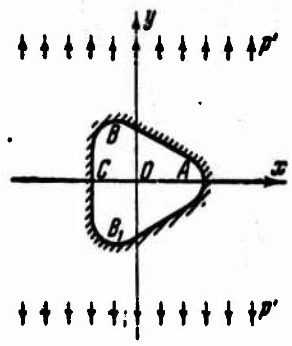

Fig. 3.

For the stress σ_θ we obtain the following expression

*See report by M. I. Nayman [4], foreign page 41, formula (67), where it is necessary to write R = α = 0, h = -p/A.

$$\sigma_\vartheta = p'\frac{A^2}{C^2} + \frac{p'}{LC^4}\{BE^4 \sin\vartheta + AC^4(\beta+\delta)\beta\delta\cos\vartheta + \\ + 2\varepsilon[BE^4\sin 2\vartheta + AC^4(\beta+\delta)\beta\delta\cos 2\vartheta] + \\ + \varepsilon^2 BC^4(\beta+\delta)\beta\delta g\sin\vartheta - \varepsilon^2 AC^4(\beta+\delta)\beta\delta[h + g(\beta+\delta)]\cos\vartheta - \\ - \varepsilon^2 BC^4(\beta+\delta)\beta\delta[kh + gl + gk(\beta+\delta)]\sin 2\vartheta + \\ + \varepsilon^2 AC^4(\beta+\delta)\beta\delta[lh + (kh+gl)(\beta+\delta) + gk(\beta^2+\delta^2+\beta\delta)]\cos 2\vartheta\} \quad (4.19)$$

It is natural to expect that maximum stress will be detained at point A:

$$\sigma_A = p' + \frac{p'}{1-2\varepsilon}\frac{\beta+\delta}{\beta\delta}\{1 + 2\varepsilon - \varepsilon^2[h + g(\beta+\delta)] - \\ - \varepsilon^3 gk\beta\delta + \varepsilon^2[h + g(\beta+\delta)][l + k(\beta+\delta)]\}. \quad (4.20)$$

From (4.19) we will obtain a formula for an isotropic plate which likewise, as in case I, coincides with the exact one*:

$$\sigma_\vartheta = \frac{p'}{C^2}(1 + 2\cos 2\vartheta - 4\varepsilon\cos\vartheta - 4\varepsilon^2). \quad (4.21)$$

Next we will introduce the calculation results for a veneer plate with an opening in which $\varepsilon = 0.25$. The stress on the contour of the opening will be presented in the form:

in case I $\quad \sigma_\vartheta = pk \quad$ (4.22)

in case II $\quad \sigma_\vartheta = p'k'. \quad$ (4.23)

In Table 1 there are given values of coefficients k at points A, C, D and values of coefficients k' at points A and C; calculated in the first, second, and third, as well as in fourth approximation. Two cases of the orientation of main directions and forces were examined: 1) elongation is made in the direction for which the Young modulus is the greatest (1.2×15^5), and 2) elongation is made in the direction for which the Young modulus has the lowest value (0.6×10^5). Two decimal signs are kept in all the numerical results. The preservation of a large number of signs is hardly necessary: it is necessary to remember that the initial data calculated values of β and δ, given at

*See [4], foreign page 41, formula (67), where it is necessary to assume $R = 0$, $\alpha = \pi/2$, $h = -p'/A$.

the end of § 2 are approximate; they are given with three significant figures.

Table 1.

Approximations	k						k'			
	$E_x = E_{max}$			$E_x = E_{min}$			$E_y = E_{max}$		$E_y = E_{min}$	
	A	D	C	A	D	C	A	C	A	C
1-e	−0.71	10.61	−0.71	−1.41	7.80	−1.41	14.38	2.49	10.45	2.05
2-e	−0.74	10.94	−0.72	−1.36	7.93	−1.40	13.95	2.34	10.27	1.99
3-e	−0.77	10.86	−0.70	−1.43	7.90	−1.37	14.09	2.30	10.33	1.97
4-e	−0.76	10.8C	−0.70	−1.39	7.88	−1.36	14.08	2.29	10.33	1.97

For comparison let us introduce stress values at the points of the opening; contour for an isotropic plate:

Case I

$$\sigma_A = \sigma_C = -p, \quad \sigma_D = 5.32p. \tag{4.24}$$

Case II

$$\sigma_A = 7p', \quad \sigma_C = 1.67p'. \tag{4.25}$$

From Table 1 it is evident that the fourth approximations within the limits of the assumed accuracy differ very little from the third approximation or do not differ at all. In order to evaluate the stress concentration close to the opening in an orthotropic plate with an accuracy sufficient for practice, it is possible to use only the third or even the second approximation. The stress concentration is greater if the plate is elongated in direction of a large Young modulus. During elongation in the direction of a smaller E, the stress changes along the contour more uniformly, as a result of which the difference between stresses in points A, C, and D is less than in the case of elongation in a direction of a larger value of E. Comparing the results derived for a veneer plate with results for an isotropic plate, we can mention that in all instances the maximum stress in a veneer plate is considerably greater than in a corresponding isotropic plate. All these conclusions are also valid for a number of other isotropic materials (wood pulp); the calculation results for them will not be given.

A solution to the problem of elongation of an orthotropic plate with a triangular opening which we indicated by the method in third approximation was first obtained by V. P. Krasyukov in his thesis*. An entirely different approximation method of solving the problem of elongating an orthotropic plate with a triangular opening and some other problems has been proposed in the K. Stephens [5] report. In this report there were given the results of calculating σ_ϑ at individual points of the opening's contour in which $\varepsilon = 0.25$ for plates with elastic constants of oak and spruce (Canadian fir).

A comparison of these results with the calculation results for the same materials based on our method shows that the numbers given in the Stephens report are nothing but our second approximations.

Stephens does not examine the other openings, and her method is in the form as it has been expounded in a report, evidently, only applicable to a plate with triangular opening.

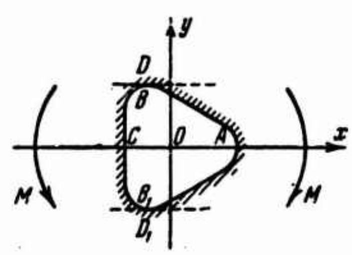

Fig. 4.

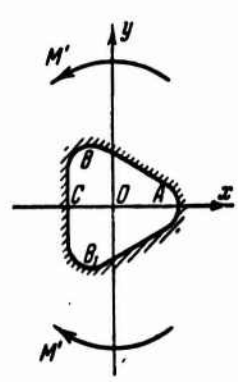

Fig. 5.

*Krasyukov, V. P. Elongation of an orthotropic plate with opening having three axes of symmetry and differing little from a circular opening. Thesis. Saratov State University, 1952.

5. ***Bending an orthotropic plate with a triangular opening by moments acting in the center plane.*** Let a rectangular orthotropic plate examined in the previous paragraph be deformed by forces distributed along the sides which are brought by moments acting in the center plane. Let us designate the values of the bending moments M and M' and the moments of inertia of the transverse sections of the plate, parallel to the sides by I and I'. Let us analyze two cases.

Case I. The forces applied to the sides of the plate, perpendicular to the axis of symmetry (Fig. 4):

$$\sigma_x^\circ = \frac{M}{I} y, \qquad \sigma_y^\circ = \tau_{xy}^\circ = 0 \tag{5.1}$$

$$\bar{\beta}_{02} = \frac{M\varepsilon^2}{8I}, \qquad \bar{\beta}_{12} = -\bar{\beta}_{13} = \frac{M\varepsilon^2}{4I}, \qquad \bar{\beta}_{04} = \frac{M\varepsilon^2}{8I}. \tag{5.2}$$

Other values of $\bar{\beta}_{km}$ and all values of $\bar{\alpha}_{km}$ are equal to zero.

$$\begin{gathered} A_{02} = -\frac{M\varepsilon^3 l}{8I(\beta-\delta)}, \qquad B_{02} = \frac{M\varepsilon^3 l}{8I(\beta-\delta)} \\ A_{01} = B_{01} = 0, \qquad A_{0m} = B_{0m} = 0 \qquad (m \geqslant 3) \end{gathered} \tag{5.3}$$

$$\begin{gathered} \sigma_\vartheta = \frac{Ma}{I}(\sin\vartheta - \varepsilon\sin 2\vartheta)\frac{B^2}{C^2} + \frac{Ma}{2ILC^3}\Big\{-BC^4(\beta+\delta)\cos 2\vartheta + AD^4\sin 2\vartheta + \\ + \varepsilon[-BC^4(\beta+\delta)(\cos\vartheta - 3\cos 3\vartheta) + AD^4(\sin\vartheta - 3\sin 3\vartheta)] + \\ + 2\varepsilon^2[-BC^4(\beta+\delta)\cos 4\vartheta + AD^4\sin 4D] + \\ + \varepsilon^3 BC^4(\beta+\delta)[h + {}^1/_2(kg\beta\delta - lh)]\cos\vartheta - \varepsilon^3 AC^4(\beta+\delta)\beta\delta g\sin\vartheta + \\ + {}^1/_2\varepsilon^3 AC^4(\beta+\delta)\beta\delta[kh + gl + gk(\beta+\delta)]\sin\vartheta\Big\}. \end{gathered} \tag{5.4}$$

In points A and C $\sigma_\vartheta = 0$. The greatest stress, just as in the case of elongation, is obtained close to point D, where the tangent is parallel to the x axis:

$$\begin{gathered} \sigma_D = \frac{Ma}{I}(\sin\vartheta_0 - \varepsilon\sin 2\vartheta_0) + \frac{Ma}{2I}\cdot\frac{\beta+\delta}{C}\Big\{-\cos 2\vartheta_0 + \varepsilon(3\cos 3\vartheta_0 - \cos\vartheta_0) - \\ - 2\varepsilon^2\cos 4\vartheta_0 + \varepsilon^3\Big[h + \frac{1}{2}(kg\beta\delta - lh)\Big]\cos\vartheta_0\Big\}. \end{gathered} \tag{5.5}$$

From (5.4) we obtain a formula for an isotropic plate coinciding with an exact one*:

$$\sigma_\vartheta = \frac{Ma}{IC^3}(\sin\vartheta - \sin 3\vartheta + \varepsilon\sin 4\vartheta - 6\varepsilon^2\sin\vartheta). \tag{5.6}$$

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*See [4], foreign page 44.

Case II. Forces applied to the sides of the plate parallel to axis of symmetry x (Fig. 5):

$$\sigma_x^\circ = 0, \qquad \sigma_y^\circ = \frac{M'}{I'}x, \qquad \tau_{xy}^\circ = 0 \tag{5.7}$$

$$\bar{\alpha}_{02} = -\frac{M'a^2}{8I'}, \quad \bar{\alpha}_{11} = \bar{\alpha}_{13} = -\frac{M'a^2}{4I'}, \quad \bar{\alpha}_{24} = -\frac{M'a^2}{8I'} \tag{5.8}$$

(the remaining values of $\bar{\alpha}_{km}$ and all values of $\bar{\beta}_{km}$ are equal to zero);

$$\begin{gathered} A_{02} = \frac{M'a^2\delta}{8I'(\beta-\delta)}, \qquad B_{02} = -\frac{M'a^2\beta}{8I'(\beta-\delta)} \\ A_{01} = B_{01} = 0, \quad A_{0m} = B_{0m} = 0 \quad (m \geqslant 3) \end{gathered} \tag{5.9}$$

$$\begin{gathered} \sigma_\vartheta = \frac{M'a}{I'}(\cos\vartheta + \varepsilon\cos 2\vartheta)\frac{A^2}{C^2} + \frac{M'a}{2I'LC^2}\{BE^4\sin 2\vartheta + AC^4(\beta+\delta)\beta\delta\cos 2\vartheta + \\ + \varepsilon[BE^4(\sin\vartheta + 3\sin 3\vartheta) + AC^4(\beta+\delta)\beta\delta(\cos\vartheta + 3\cos 3\vartheta)] + \\ + 2\varepsilon^2[BE^4\sin 4\vartheta + AC^4(\beta+\delta)\beta\delta\cos 4\vartheta] + \\ + {}^1/_2\,\varepsilon^3BC^4(\beta+\delta)\beta\delta[2g - kh - gl - gk(\beta+\delta)]\sin\vartheta + \\ + {}^1/_2\,\varepsilon^2AC^4(\beta+\delta)\beta\delta[h(l-2) + (kh+gl-2g)(\beta+\delta) - \\ - gk\beta\delta + gk(\beta+\delta)^2]\cos\vartheta\}. \end{gathered} \tag{5.10}$$

At point A

$$\begin{gathered} \sigma_A = \frac{M'a}{I'}(1+\varepsilon) + \frac{M'a}{I'} \qquad \frac{1}{1-2\varepsilon}\,\frac{\beta+\delta}{\beta\delta}\{0.5 + 2\varepsilon + \varepsilon^2 + \\ + \varepsilon^2 0{,}25[h(l-2) + (kh+gl-2g)(\beta+\delta) - gk\beta\delta + gk(\beta+\delta)^2]\}. \end{gathered} \tag{5.11}$$

It is highly possible that this stress value will be greatest on the contour of the opening. We get the stress in point C by the same formula (5.11), substituting M' by -M' and ε by -ε.

For an isotropic plate

$$\sigma_\vartheta = \frac{M'a}{I'C^2}[\cos\vartheta + \cos 3\vartheta + \varepsilon(\cos 4\vartheta - 2) - 6\varepsilon^2\cos\vartheta]. \tag{5.12}$$

This formula also coincides with the exact one*.

Let us represent stress on the contour of the opening caused by moments M' and M, in the following manner:

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*See [4], foreign page 42 (in formula for σ_ϑ, placed on the page it is necessary to assume $\alpha = \pi/2$, $AR = -M'/I'$).

$$\sigma_\vartheta = \frac{Ma}{I} k_1 \tag{5.13}$$

$$\sigma_\vartheta = \frac{M'a}{I'} k_1' . \tag{5.14}$$

In Table 2 there are given values of coefficients k_1 at point D and k_1' in points A and C for a veneer plate; formerly, parameter ε was assumed to be equal to 0.25. For every point there are given four approximations. Cases are investigated when the greatest Young modulus corresponds to the direction of the x-axis, even when the least value of E corresponds to it.

Table 2.

Approximations	k_1		k_1'			
	$E_x=E_{max}$	$E_x=E_{min}$	$E_y=E_{max}$		$E_y=E_{min}$	
	D	D	A	C	A	C
1-e	6.77	5.11	10.16	–0.75	7.56	–0.75
2-e	6.73	5.09	10.71	–0.94	7.95	–0.88
3-e	6.71	5.08	10.70	–0.94	7.94	–0.88
4-e	6.69	5.07	10.70	–0.94	7.94	–0.88

For an isotropic plate with the same opening we have:

Case 1

$$\sigma_D = 3.63 \frac{Ma}{I} . \tag{5.15}$$

Case 2

$$\sigma_A = 5.50 \frac{M'a}{I'}, \qquad \sigma_C = -0.83 \frac{M'a}{I'} . \tag{5.16}$$

As can be seen from Table 2, for the evaluation of the stress concentration is sufficient only for a second approximation. Just as in the case of elongation, the concentration of stresses in an anisotropic plate is obtained considerably greater than in an isotropic plate.

6. *Determining stresses in a plate with an oval opening.* Let us deduce an approximate solution to the problem about an elastic equilibrium of a plate with an opening where the contour is given by the equations

$$x = a(\cos\vartheta + \varepsilon\cos 3\vartheta), \quad y = a(c\sin\vartheta - \varepsilon\sin 3\vartheta) \quad (0 < c < 1) . \tag{6.1}$$

We assume, as before, that the main vector of forces X_n and Y_n, distributed along the edge of the opening, equals zero. Let us limit ourselves only to the second approximation, i.e., let us assume

$$\Phi_1 = \Phi_{10} + \varepsilon\Phi_{11} + \varepsilon^2\Phi_{12}, \qquad \Phi_2 = \Phi_{20} + \varepsilon\Phi_{21} + \varepsilon^2\Phi_{22}. \tag{6.2}$$

Functions Φ_{1k} and Φ_{2k} are expressed through functions $f_{1k}(\zeta_1)$ and $f_{2k}(\zeta_2)$ by formulas (1.9), where it is necessary to assume N = 3. At $c \neq 1$ the relationship between variables ζ_1, ζ_2, and ζ, $\bar{\zeta}$ is given by relationship (1.7). Functions f_{1k} and f_{2k} have the form of (1.14), and on the contour of the opening values of the order of (1.15) are assumed.

Let us write out the boundary values Φ_{1k} and Φ_{2k} (k = 0, 1, 2), which are obtained on the basis of formulas (1.9), (2.9), and (2.10).

$$\Phi_{10} = A_0 + \sum_{m=1}^{\infty} A_{0m}\sigma^{-m}, \qquad \Phi_{20} = B_0 + \sum_{m=1}^{\infty} B_{0m}\sigma^{-m} \tag{6.3}$$

$$\begin{gathered}\Phi_{11} = A_1 - A_{01}\beta_1\sigma + \sum_{m=1}^{\infty} (A_{1m} + A_{0m}^1)\sigma^{-m}, \\ \Phi_{21} = B_1 - B_{01}\beta_2\sigma + \sum_{m=1}^{\infty} (B_{1m} + B_{0m}^1)\sigma^{-m}\end{gathered} \tag{6.4}$$

$$\begin{aligned}\Phi_{12} = {} & A_2 + [(3A_{01}\alpha_1 + 6A_{02})\beta_1^2 - A_{11}\beta_1]\sigma + \\ & + 3A_{02}\beta_1^2\sigma^2 + A_{01}\beta_1^2\sigma^2 + \sum_{m=1}^{\infty} (A_{2m} + A_{1m}^1 + A_{0m}^2)\sigma^{-m}, \\ \Phi_{22} = {} & B_2 + [(3B_{01}\alpha_2 + 6B_{02})\beta_2^2 - B_{11}\beta_2]\sigma + \\ & + 3B_{02}\beta_2^2\sigma^2 + B_{01}\beta_2^2\sigma^2 + \sum_{m=1}^{\infty} (B_{2m} + B_{1m}^1 + B_{0m}^2)\sigma^{-m}.\end{aligned} \tag{6.5}$$

Satisfying boundary conditions (1.11), where k = 0, 1, 2, we obtain equations for the coefficients of functions convergent with equations (3.9)-(3.11). Without writing them out, let us give only finite expressions for boundary values Φ_{10}, Φ_{11}, and Φ_{12}:

$$\Phi_{10} = A_0 + \sum_{m=1}^{\infty} \frac{\bar{\beta}_{0m} - \mu_2\bar{\alpha}_{0m}}{\mu_1 - \mu_2}\sigma^{-m} \tag{6.6}$$

$$\Phi_{11} = A_1 + \sum_{m=0}^{\infty} \frac{\bar{\beta}_{1m} - \mu_2\bar{\alpha}_{1m}}{\mu_1 - \mu_2}\sigma^{-m} - A_{01}\beta_1\sigma - \frac{\bar{A}_{01}\bar{\beta}_1(\mu_2 - \bar{\mu}_1) + \bar{B}_{01}\bar{\beta}_2(\mu_2 - \bar{\mu}_2)}{\mu_1 - \mu_2}\,\frac{1}{\sigma} \tag{6.7}$$

$$\Phi_{12} = A_2 + \sum_{m=1}^{\infty} \frac{\bar{\beta}_{2m} - \mu_2 \bar{\alpha}_{2m}}{\mu_1 - \mu_2} \sigma^{-m} + [(3A_{01}\alpha_1 + 6A_{03})\beta_1^2 - A_{11}\beta_1]\sigma + 3A_{03}\beta_1^2\sigma^2 + A_{01}\beta_1^3\sigma^3 + \left\{[(3\bar{A}_{01}\bar{\alpha}_1 + 6\bar{A}_{03})\bar{\beta}_1^2 - \bar{A}_{11}\bar{\beta}_1] \frac{\mu_2 - \bar{\mu}_1}{\mu_1 - \mu_2} + [(3\bar{B}_{01}\bar{\alpha}_2 + 6\bar{B}_{03})\bar{\beta}_2^2 - \bar{B}_{11}\bar{\beta}_2] \frac{\mu_2 - \bar{\mu}_2}{\mu_1 - \mu_2}\right\} \frac{1}{\sigma} + 3 \frac{\bar{A}_{03}\bar{\beta}_1^2(\mu_2 - \bar{\mu}_1) + \bar{B}_{03}\bar{\beta}_2^2(\mu_2 - \bar{\mu}_2)}{\mu_1 - \mu_2} \frac{1}{\sigma^2} + \frac{\bar{A}_{01}\bar{\beta}_1^3(\mu_2 - \bar{\mu}_1) + \bar{B}_{01}\bar{\beta}_2^3(\mu_2 - \bar{\mu}_2)}{\mu_1 - \mu_2} \frac{1}{\sigma^3}. \quad (6.8)$$

Coefficients A_{0m} and B_{0m} are determined by formulas (1.16). The boundary values of functions Φ_{20}, Φ_{21}, and Φ_{22} will be derived from (6.6)-(6.8) with tbe substitution of A, B, μ_1, μ_2, β_1, β_2, α_1, α_2 everywhere, values B, A. μ_2, μ_1, β_2, β_1, α_2, α_1 respectively. Coefficients A_{11} and B_{11}, which must be known for the purpose of calculating the second approximation, will be determined by formula

$$A_{11} = \frac{\bar{\beta}_{11} - \mu_2 \bar{\alpha}_{11}}{\mu_1 - \mu_2} - \frac{\bar{A}_{01}\bar{\beta}_1(\mu_2 - \bar{\mu}_1) + \bar{B}_{01}\bar{\beta}_2(\mu_2 - \bar{\mu}_2)}{\mu_1 - \mu_2} + (A_{01}\alpha_1 + 3A_{03})\beta_1. \quad (6.9)$$

and by a similar one. The constant components A_k and B_k can be considered arbitrary.

Formula (2.7) will serve for calculating stress σ_ϑ at the edge of the opening, or if the plate is orthotropic, (2.16) will serve where it is necessary to write

$$A = c\cos\vartheta - 3\varepsilon\cos 3\vartheta, \quad B = c\sin\vartheta + 3\varepsilon\sin 3\vartheta, \quad C^2 = A^2 + B^2. \quad (6.10)$$

7. *Elongation of an orthotropic plate with oval opening.* Based on the general solution given in § 6, we can easily obtain approximate formulas for stress calculation in a rectangular plate, weakened by an opening of type (6.1), elongated by normal forces p and p', uniformly distributed over the sides.

Without considering the problem in its entire generality, let us assume the following limitations: 1) the dimensions of the opening are small in comparison with the dimensions of the plate, and its center coincides with the center of the plate; 2) the plate is orthotropic, whereas the main directions of elasticity in it are parallel to the sides and the opening is cut out so that its axes of symmetry are parallel to the sides of the plate (and consequently,

to the main directions of elasticity also). Let us place the origin of the coordinates in the center of the opening, and let us direct the x axis along its greater axis. The solution of the problem is obtained by the same method as for a plate with a triangular opening. Therefore, we will give only the basic results, omitting all conclusions and details.

It is possible to distinguish two basic cases.

Case I. Elongation in a direction of the greater axis of the opening (Fig. 6)

$$\sigma_x^{\circ} = p, \qquad \sigma_y^{\circ} = \tau_{xy}^{\circ} = 0 \tag{7.1}$$

$$\bar{\beta}_{01} = -\frac{paci}{2}, \qquad \bar{\beta}_{13} = \frac{pai}{2} \tag{7.2}$$

the remaining values of $\bar{\beta}_{km}$ and all values of $\bar{\alpha}_{km}$ equal zero;

$$A_{01} = -\frac{pac}{2(\beta-\delta)} \qquad B_{01} = \frac{pac}{2(\beta-\delta)}, \qquad A_{0m} = B_{0m} = 0 \qquad (m > 2). \tag{7.3}$$

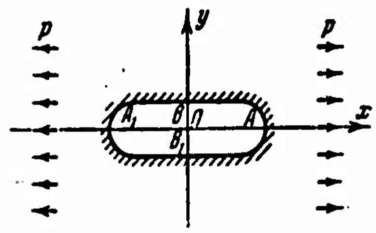

Fig. 6.

The formula for the determining stress σ_ϑ at the edge of the opening in the second approximation emanating from (2.16) has the form*:

$$\begin{aligned}\sigma_\vartheta = p\frac{R^2}{C^2} + \frac{p}{LC^3}\{c[AD^4\cos\vartheta + BC^4(\beta+\delta)\sin\vartheta] + \\ + \varepsilon[2AC^4c(\beta+\delta)\beta\delta b_{11}\cos\vartheta - 3AD^4\cos 3\vartheta - BC^4(\beta+\delta)(2ca_{11}\sin\vartheta + \\ + 3\sin 3\vartheta)] + 2\varepsilon^2 C^4 c(\beta+\delta)[-A\beta\delta(b_{21}\cos\vartheta + 3b_{23}\cos 3\vartheta) + \\ + B(a_{21}\sin\vartheta + 3a_{23}\sin 3\vartheta)]\}.\end{aligned} \tag{7.4}$$

The distribution of stresses along the contour of the opening will be symmetrical with respect to the x and y axes. The greatest stress will be derived at points A, A_1 or B, B_1 (Fig. 6).

*See designations (6.10) and (2.19), (2.20).

In points A, A_1

$$\sigma_A = -\frac{p}{c-3\varepsilon}\frac{1}{\beta\delta}\{c-\varepsilon[3+2(\beta+\delta)cb_{11}]+2\varepsilon^2(\beta+\delta)c(b_{31}+3b_{33})\}. \tag{7.5}$$

In points B, B_1

$$\sigma_B = p+\frac{p}{1-3\varepsilon}(\beta+\delta)[c+\varepsilon(3-2ca_{11})+2\varepsilon^2c(a_{31}-3a_{33})]. \tag{7.6}$$

For an isotropic plate at the same points we obtain

$$\sigma_A = -\frac{p}{c-3\varepsilon}\left[c-\varepsilon\frac{3-c}{1+c}+\varepsilon^2\frac{8c}{(1+c)^2}\right] \tag{7.7}$$

$$\sigma_B = p+\frac{2p}{1-3\varepsilon}\left[c+\varepsilon\frac{3+5c}{1+c}+\varepsilon^2\frac{4c}{(1+c)^2}\right]. \tag{7.8}$$

In contrast to the plate with a triangular opening these formulas do not coincide with exact ones, but are approximate [to the very same extent as formulas (7.4)-(7.6)].

At c = 1 from (7.4)-(7.8) we will obtain formulas for the elongated plate with opening, close to a square*.

Case II. Elongation in the direction of the small azis of the opening (Fig. 7):

$$\sigma_x{}^\circ=0, \quad \sigma_y{}^\circ=p', \quad \tau_{xy}{}^\circ=0 \tag{7.9}$$

$$\bar{\alpha}_{01}=\bar{\alpha}_{13}=-\frac{p'a}{2} \tag{7.10}$$

the remaining values of $\bar{\alpha}_{km}$ and all values of $\bar{\beta}_{km}$ are equal to zero;

$$A_{01}=\frac{p'a\delta}{2(\beta-\delta)}, \quad B_{01}=-\frac{p'a\beta}{2(\beta-\delta)}, \quad A_{m0}=B_{0m}=0 \quad (m>2) \tag{7.11}$$

$$\begin{aligned}\sigma_\vartheta = p'\frac{A^2}{C^2}+\frac{p'}{LC^3}\{AC^4(\beta+\delta)\beta\delta\cos\vartheta+BE^4\sin\vartheta+\\ +\varepsilon[AC^4(\beta+\delta)\beta\delta(-2c_{11}\cos\vartheta+3\cos3\vartheta)+2BC^4(\beta+\delta)\beta\delta b_{11}\sin\vartheta+\\ +3BE^4\sin3\vartheta]+2\varepsilon^2C^4(\beta+\delta)\beta\delta[A(c_{31}\cos\vartheta+3c_{33}\cos3\vartheta)-\\ -B(b_{31}\sin\vartheta+3b_{33}\sin3\vartheta)]\}.\end{aligned} \tag{7.12}$$

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*See [1], § 5.

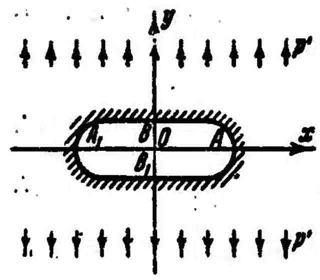

Fig. 7.

At points A and A_1 (Fig. 7)

$$\sigma_A = p' + \frac{p'}{c - 3\varepsilon} \frac{\beta + \delta}{\beta\delta} [1 + \varepsilon (3 - 2c_{11}) + 2\varepsilon^2 (c_{21} + 3c_{22})] . \tag{7.13}$$

At points B and B_1

$$\sigma_B = -\frac{p'}{1 - 3\varepsilon} \beta\delta \{1 - \varepsilon [3 + 2 \ (\beta + \delta) b_{11}] + 2\varepsilon^2 (\beta + \delta)(b_{21} - 3b_{22})\} . \tag{7.14}$$

One of these two values will be the greatest (probably, the first one; but the possibility is not excluded that the greatest will be not σ_A, but σ_B).

At these points of the isotropic plate

$$\sigma_A = p' + \frac{2p'}{c - 3\varepsilon} \left[1 + \varepsilon \frac{5 + 3c}{1 + c} + \varepsilon^2 \frac{4}{(1 + c)^2}\right] \tag{7.15}$$

$$\sigma_B = -\frac{p'}{1 - 3\varepsilon} \left[1 + \varepsilon \frac{1 - 3c}{1 + c} + \varepsilon^2 \frac{8}{(1 + c)^2}\right] . \tag{7.16}$$

As an illustration let us give the calculation results for a veneer plate with an opening of type (6.1) elongated in the direction of large or small axis of the opening. We consider two openings.

The first is characterized by parameters $c = 0.537$, $\varepsilon = -0.038$, and is a peculiar oval in which the length to width ratio equals 1.93. The problem of elongating an isotropic plate with such an opening was considered by Greenspan [6], and the problem of bending by moments was considered by Joseph and Brock [7].

The second opening is characterized by parameters $c = 0.36$, $\varepsilon = -0.04$, and in form it is close to a rectangle in which the short sides are replaced by semicircles. The ratio of length of this

opening to its width equals three; the curvature in the center of its long sides equals zero.

Formulas for stresses at the edge of the opening will be recorded in such a manner:

$$\sigma_\vartheta = pk \qquad (7.17)$$

$$\sigma_\vartheta = p'k'. \qquad (7.18)$$

In Table 3 there are given values of coefficients k and k' at points A and B, found in the first and second approximations for a veneer plate weakened by an opening with parameters c = 0.537 and ε = -0.038. In this table and in the one following two significant decimal points are kept. Instances of elongation in a direction for which the Young modulus is greatest, and in a direction for which the Young modulus is smallest, are examined.

Table 3. c = 0.537, ε = -0.038

Approximations	k				k'			
	$E_x = E_{max}$		$E_x = E_{min}$		$E_y = E_{max}$		$E_y = E_{min}$	
	A	B	A	B	A	B	A	B
1-e	—0.63	2.51	—1.27	2.10	6.65	—0.63	5.04	—1.24
2-e	—0.63	2.50	—1.27	2.09	6.62	—0.63	5.03	—1.24

In an isotropic plate with such an opening, the following stresses are obtained.

During elongation in the direction of great axis of the opening

$$\sigma_A = -0.92p, \quad \sigma_B = -1.71p. \qquad (7.19)$$

During elongation in the direction of the small axis

$$\sigma_A = -0.92p, \quad \sigma_B = 1.39p. \qquad (7.20)$$

In Table 4 there are given values k and k' at points A and B for a veneer plate with an opening in which c = 0.36 and ε = -0.04.

Table 4.

Approximations	k				k'			
	$E_x = E_{max}$		$E_x = E_{min}$		$E_y = E_{max}$		$E_y = E_{min}$	
	A	B	A	B	A	B	A	B
1-e	−0.62	1.79	−1.25	1.59	8.54	−0.61	6.39	−4.20
2-e	−0.63	1.78	−1.24	1.58	8.50	−0.61	6.38	−4.19

For such an isotropic plate we obtain:

during elongation in the direction of the great axis of the opening

$$\sigma_A = 3.58p', \quad \sigma_B = -0.92p' \tag{7.21}$$

during elongation in the direction of the small axis

$$\sigma_A = 4.44p', \quad \sigma_B = -0.90p'. \tag{7.22}$$

Calculations show that when two significant decimal places are kept, the third approximations in all the examined cases do not differ from the second, but the second approximations also differ very little from the first or do not differ at all. The stress concentration is greater in the case when the plate is elongated in the direction for which the Young modulus has its maximum value. The greatest stress in a veneer plate elongated in its main direction is greater in all instances than in the same isotropic plate.

8. Bending of an orthotropic plate with an oval opening by moments acting in the center plane. Let us examine that on the plate weakened by an oval opening examined in a previous paragraph, there act forces leading to moments M and M'. Here let us also examine two basic cases.

Case 1. Forces applied to the sides of the plate parallel to the small axis of the opening (Fig. 8).

$$\sigma_x^0 = \frac{M}{I}y, \qquad \sigma_y^0 = \tau_{xy}^0 = 0 \tag{8.1}$$

$$\bar{\beta}_{02} = \frac{Ma^2c^2}{8I}, \qquad \bar{\beta}_{12} = -\bar{\beta}_{14} = \frac{Ma^2c}{4I}, \qquad \bar{\beta}_{26} = \frac{Ma^2}{8I} \tag{8.2}$$

$\bar{\beta}_{km} = 0$ for other values of k, m; $\bar{\alpha}_{km} = 0$ for all values of k and m;

$$A_{03} = -\frac{Me^2c^3l}{8I(\beta-\delta)}, \qquad B_{03} = \frac{Me^2c^3i}{8I(\beta-\delta)}$$
$$A_{01} = B_{01} = 0, \qquad A_{0m} = B_{0m} = 0 \qquad (m > 3) \tag{8.3}$$

$$\sigma_\vartheta = \frac{Ma}{I}(c\sin\vartheta - \varepsilon\sin 3\vartheta)\frac{B^3}{C^3} + \frac{Ma}{2ILC^3}\{c^3[-BC^4(\beta+\delta)\cos 2\vartheta + AD^4\sin 2\vartheta] + 2\varepsilon c[-BC^4(\beta+\delta)(\cos 2\vartheta - 2\cos 4\vartheta) + AD^4(\sin 2\vartheta - 2\sin 4\vartheta)] + 3\varepsilon^2[BC^4(\beta+\delta)(2c^3a_{23}\cos 2\vartheta - \cos 6\vartheta) + 2AC^4c^3(\beta+\delta)\beta\delta b_{23}\sin 2\vartheta + AD^4\sin 6\vartheta]\} \tag{8.4}$$

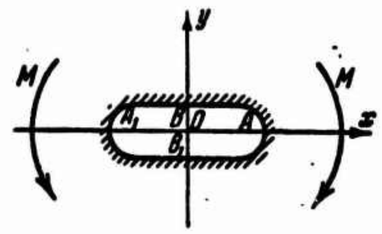

Fig. 8.

Values A, B, and C^2 are determined by formulas (6.10), and a_{23}, b_{23} are determined by formulas (2.19).

At points A and A_1 at the ends of the great axis (Fig. 8) $\sigma_\vartheta = 0$.

At point B we obtain the greatest stress:

$$\sigma_B = \frac{Ma}{I}\left\{c + \varepsilon + \frac{\beta+\delta}{1-3\varepsilon}[0.5c^3 + 3\varepsilon c + 1.5\varepsilon^2(1 - 2c^3a_{23})]\right\}. \tag{8.5}$$

At opposite point B_1 the stress has the same value but is of opposite sign.

For the isotropic plate

$$\sigma_B = \frac{Ma}{I(1-3\varepsilon)}[c(1+c) + \varepsilon(1+3c)]. \tag{8.6}$$

The latter formula coincides with the precise formula*.

---

*It is derived from a formula given in report [7], if a changeover is made to our designations.

Case 2. The forces applied to the sides of the plate parallel to the large axis of the opening (Fig. 9):

$$\sigma_x^{\circ} = 0, \qquad \sigma_y^{\circ} = \frac{M'}{I'} x, \qquad \tau_{xy}^{\circ} = 0 \tag{8.7}$$

$$\bar{\alpha}_{02} = -\frac{M'a^3}{8I'}, \qquad \bar{\alpha}_{12} = \bar{\alpha}_{14} = -\frac{M'a^3}{4I'}, \qquad \bar{\alpha}_{26} = -\frac{M'a^3}{8I'}. \tag{8.8}$$

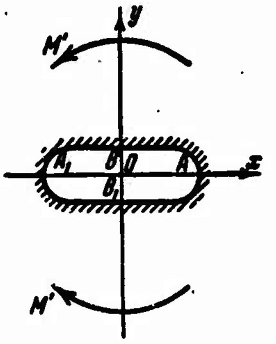

Fig. 9.

The values $\bar{\alpha}_{km} = 0$ for other values of k, m; $\bar{\beta}_{km} = \theta$ for all values of k, m;

$$A_{02} = \frac{M'a^3\delta}{8I'(\beta-\delta)}, \qquad B_{02} = -\frac{M'a^3\beta}{8I'(\beta-\delta)}$$
$$A_{01} = B_{01} = 0, \qquad A_{0m} = B_{0m} = 0 \qquad (m \geq 3) \tag{8.9}$$

$$\begin{aligned}\sigma_\vartheta = \frac{M'a}{I'}(\cos\vartheta + \varepsilon\cos 3\vartheta)\frac{A^2}{C^2} + \frac{M'a}{2I'LC^2}\{AC^4(\beta+\delta)\beta\delta\cos 2\vartheta + \\ + BE^4\sin 2\vartheta + 2\varepsilon[AC^4(\beta+\delta)\beta\delta(\cos 2\vartheta + 2\cos 4\vartheta) + BE^4(\sin 2\vartheta + \\ + 2\sin 4\vartheta)] + 3\varepsilon^2[AC^4(\beta+\delta)\beta\delta(2c_{23}\cos 2\vartheta + \cos 6\vartheta) - \\ - 2BC^4(\beta+\delta)\beta\delta b_{23}\sin 2\vartheta + BE^4\sin 6\vartheta]\}.\end{aligned} \tag{8.10}$$

At points B and B_1

$$\sigma_\vartheta = 0.$$

At poi:.: A

$$\sigma_A = \frac{M'a}{I'}(1+\varepsilon) + \frac{M'a}{I'}\cdot\frac{1}{c-3\varepsilon}\cdot\frac{\beta+\delta}{\beta\delta}[0.5 + 3\varepsilon + 1{,}5\varepsilon^2(1+2c_{23})]. \tag{8.11}$$

This value of σ_ϑ will be greatest. At the same point of the isotropic plate

$$\sigma_A = \frac{M'a}{I'(c-3\varepsilon)}[1 + c + \varepsilon(3+c)]. \tag{8.12}$$

This formula is also identical with the exact one.

The stress close to the contour of the opening, just as before, is in the form

$$\sigma_\phi = \frac{Ma}{I} k_1 \qquad (8.13)$$

or

$$\sigma_\phi = \frac{M'a}{I'} k_1'. \qquad (8.14)$$

In Table 5 there are given values of coefficient k_1 at point B and of coefficient k_1' at point A for a veneer plate with an opening in which c = 0.537 and ε = -0.038. For such an isotropic plate we obtain:

case 1

$$\sigma_B = 0.65 \frac{Ma}{I} \qquad (8.17)$$

case 2

$$\sigma_A = 2.16 \frac{M''a}{I'}. \qquad (8.18)$$

In Table 6 there are given values of coefficients k_1 at point B and of k_1' in point A for a veneer plate weakened by opening with parameters c = 0.36 and ε = -0.04.

In an isotropic plate with such an opening we obtain:

case 1

$$\sigma_B = 0.36 \frac{Ma}{I} \qquad (8.17)$$

case 2

$$\sigma_A = 2.55 \frac{M'a}{I'}, \qquad (8.18)$$

In both tables there are given coefficients calculated in first and second approximations. Two decimal places have been everywhere.

Table 5.

$c = 0{,}537,\ \varepsilon = -0.038$

Approximations	$k_1\,(B)$		$k_1'\,(A)$	
	$E_x = E_{max}$	$E_x = E_{min}$	$E_y = E_{max}$	$E_y = E_{min}$
1-e	0.84	0.74	3.61	2.84
2-e	0.84	0.74	3.60	2.84

Table 6.

$c = 0.36,\ \varepsilon = -0.04$

Approximations	$k'\,(B)$		$k_1'\,(A)$	
	$E_x = E_{max}$	$E_x = E_{min}$	$E_y = E_{max}$	$E_y = E_{min}$
1-e	0.41	0.38	4.50	3.47
2-e	0.41	0.39	4.49	3.46

If two decimal places are kept, the third approximations will not differ from the second at all, and the second approximations will differ very little from the first or coincide with the first ones. Thus, during the bending by moments of veneer plates with the indicated openings, the first approximation already assures an accuracy totally adequate for practice for the evaluation of the stress concentration. Remarks with respect to the concentration of stresses made previously, in § 7, also pertain to the same extent to the case of bending.

An exact solution for an anisotropic plate with an opening which is neither circular nor elliptical has as yet not been found. An approximate method offers the possibility of calculating the stresses in anisotropic plates close to all possible openings of type (1.1) and thus by means of a relatively noncomplex mathematical apparatus with an accuracy sufficient for practice. We assume that the given numerical examples sufficiently convincingly attest the effectiveness of this method (even in such cases when the openings differ quite considerably from elliptical and circular openings).

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ABSTRACT

(U) The author has in a previous paper [Inzen. Sb. 17(1953), 3-28; MR 16, 540] developed a method for an approximative solution of plane elastic case relative to an anistropic plate, weakened through an opening slightly different from the circular form. He has analyzed the distrubution of stresses in the neighbourhood of an opening with four axes of symmetry in the cases of the extension and of bending generated through moments. In the present paper he applies this method to solution of the plane case for an anisotropic plate with an opening slightly different from the ellipitic form. He gives the approximative solutions for the case of an opening in an orthotropic plate which is near to an equilateral triangle with rounded off corners, and to an oval opening (more exactly, to an opening of the rectangular form with curved shorter sides). The general case of distribution of external forces is considered as well as two particular cases: of extension and of bending through moments.